# Simple Harmonic Motion

Lots of things vibrate or oscillate. A vibrating tuning fork, a moving child's playground swing, and the loudspeaker in a radio are all examples of physical vibrations. There are also electrical and acoustical vibrations, such as radio signals and the sound you get when blowing across the top of an open bottle.

One simple system that vibrates is a mass hanging from a spring. The force applied by an ideal spring is proportional to how much it is stretched or compressed. Given this force behavior, the up and down motion of the mass is called *simple harmonic* and the position can be modeled with

$$y = A\cos(2\pi ft + \phi)$$

In this equation, $y$ is the vertical displacement from the equilibrium position, $A$ is the amplitude of the motion, $f$ is the frequency of the oscillation, $t$ is the time, and $\phi$ is a phase constant. This experiment will clarify each of these terms.

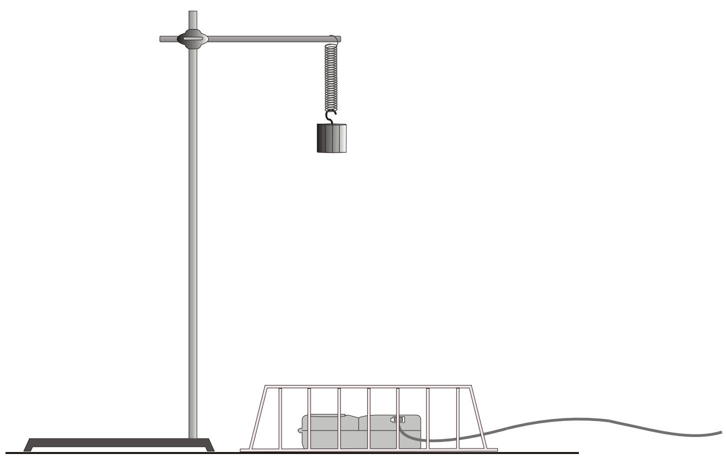

*Figure 1*

## OBJECTIVES

- Measure the position and velocity as a function of time for an oscillating mass and spring system.
- Compare the observed motion of a mass and spring system to a mathematical model of simple harmonic motion.
- Determine the amplitude, period, and phase constant of the observed simple harmonic motion.

## MATERIALS

TI-Nspire handheld **or**
computer and TI-Nspire software
CBR 2 **or** Go! Motion, **or**
Motion Detector and data-collection interface
ring stand, rod, and right angle clamp
wire basket
200 g and 300 g masses
spring, with a spring constant of approximately 10 N/m
zip ties

  

## PRELIMINARY QUESTIONS

1. Attach the 200 g mass to the spring and hold the free end of the spring in your hand, so the mass and spring hang down with the mass at rest. Lift the mass about 10 cm and release. Observe the motion. Sketch a graph of position *vs*. time for the mass.

2. Just below the graph of position *vs*. time, and using the same length time scale, sketch a graph of velocity *vs*. time for the mass.

## PROCEDURE

1. Place the Motion Detector about 50 cm below the mass. Make sure there are no objects near the path between the detector and mass, such as a table edge. Place the wire basket over the Motion Detector to protect it.

2. Attach the spring to a horizontal rod connected to the ring stand and hang the mass from the spring as shown in Figure 1. Securely fasten the 200 g mass to the spring and the spring to the rod using zip ties so the mass cannot fall.

3. If your Motion Detector has a switch, set it to Normal. Connect the Motion Detector to the data-collection interface. Connect the interface to the TI-Nspire handheld or computer. (If you are using a CBR 2 or Go! Motion, you do not need a data-collection interface.)

4. Be sure your handheld or computer software is set to perform angle calculations in Radians.

5. Choose New Experiment from the Experiment menu. For this experiment, the default data-collection parameters for a Motion Detector will be used (Rate: 20 samples per second; Duration: 5 seconds).

6. Click the Graph View tab ( ). Choose Show Graph ► Graph 1 from the Graph menu. Only the Position vs. Time Graph will be displayed.

7. With the spring in its resting position, zero the motion detector by choosing Set Up Sensors ► Zero from the Experiment menu. The values should be close to zero.

8. Make a preliminary run to make sure things are set up correctly. Lift the mass upward about five centimeters and release. The mass should oscillate along a vertical line only, and should never come closer than 15 cm to the Motion Detector. Start data collection ( ).

9. After five seconds, data collection will stop. The position graph should show a clean sinusoidal curve. If it has flat regions or spikes, reposition the Motion Detector and try again.

10. Measure the time interval between adjacent maximum positions. This is the *period*, *T*, of the motion. The frequency, *f*, is the reciprocal of the period, $f = 1/T$. Based on your period measurement, calculate the frequency. Record the period and frequency of this motion in Table 1.

11. The amplitude, *A*, of simple harmonic motion is the maximum distance from the equilibrium position. Estimate values for the amplitude from your position graph. Enter the values in Table 1.

12. Click the Store Latest Data Set button ( ) to save the first run. Repeat Steps 8–11 with the same 200 g mass, with an amplitude greater than 5 cm, but less than 10 cm.

13. Click the Store Latest Data Set button (icon) to save the second run. Change the mass to 300 g and repeat Steps 7–11. Use an amplitude of about 5 cm for this trial.

## DATA

| Table 1 | | | | |
|---|---|---|---|---|
| Run | *Mass* (g) | *A* (m) | *T* (s) | *f* (Hz) |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |

| Model equation with parameters | |
|---|---|

## PROCESSING THE DATA

1. You can compare your experimental data to the sinusoidal function model. Try it with your 300 g data. To model this relationship, you will use the model

$$Y = \mathbf{a} * \cos(2\pi * \mathbf{b} * (x - \mathbf{c})) + \mathbf{d}$$

Comparing terms, listing the textbook model's terms first, the amplitude *A* corresponds to the parameter ***a***, *f* corresponds to **b**, $\phi$, the phase shift, corresponds to $-2\pi$***b** ***c**. The time, *t*, is represented by the variable *x*. The textbook model gives the displacement from equilibrium. Although you zeroed the motion detector while in the equilibrium position, the constant **d** will allow you to adjust for errors.

   a. Prepare to model your data by choosing Model from the Analyze menu.
   b. Type the equation a∗cos(2π∗b∗(x – c)) + d.
   c. Select OK.
   d. Enter the amplitude you measured for run3 for **a**. Adjust the spin increment for **a** to 0.001.
   e. Enter the frequency in Hz you determined for run3 for **b**. Adjust the spin increment for **b** to 0.001.
   f. Enter the *time* that corresponds to the first peak of the position data for **c**. Adjust the spin increment for **c** to 0.001.
   g. Enter 0 for the initial value for **d**. Adjust the spin increment for **d** to 0.001. Select OK.

2. Adjust the model until it matches closely the experimental data.

   a. The parameter values are located to the left of the graph in the Graph View details box. Adjust the **c** and **d** values until your model comes very close to the experimental data. You may need to adjust **a** and **b** slightly from your measured values. Continue to adjust the values until the model matches very closely with the experimental data.
   b. Record the model equation in the data table.

3. Choose Show Graph ► Graph 2 from the Graph menu. This will show the Velocity graph. Using data for the 300-g mass, use Data Markers to mark the times at which the velocity is the greatest.

a. To add a data marker, click on the graph and use ► and ◄ to locate the points.
b. Move your cursor over the Graph View details box and launch the contextual menu (handheld – /b; computer – right-click). Select the Add Data Marker option.
c. Choose Show Graph ► Graph 2 from the Graph menu. This will show both the position and velocity graphs for the data. Examine the marked points and look for any relationships between position and velocity when velocity is a maximum.

4. Repeat Step 3 above this time identifying the points where the velocity is a minimum.
5. Repeat Step 3 this time identifying the points where the velocity is zero.

## QUESTIONS

1. Compare your position graphs to your sketched prediction in the Pre-Lab Questions. How are the graphs similar? How are they different?
2. Compare your velocity graphs to your sketched prediction in the Pre-Lab Questions. How are the graphs similar? How are they different?
3. Relative to the equilibrium position, where is the mass when the velocity is zero? Where is the mass when the velocity is greatest?
4. View the graphs of the last run. Compare the position *vs*. time and the velocity *vs*. time graphs. How are they the same? How are they different?
5. Does the frequency, *f*, appear to depend on the amplitude of the motion? Do you have enough data to draw a firm conclusion?
6. Does the frequency, *f*, appear to depend on the mass used? Did it change much in your tests?
7. Does the model fit the data well? How can you tell?
8. Predict what would happen to the plot of the model if you doubled the parameter for **a** (the amplitude) by sketching both the current model and the new model with doubled **a**. Double the parameter for **a** to test your prediction.
9. Predict how the model plot would change if you doubled **b**, and then check by modifying the model definition.

## EXTENSIONS

1. Investigate how changing the spring amplitude changes the period of the motion. Take care not to use a large amplitude so that the mass does not come closer than 15 cm to the detector or fall from the spring.
2. How will *damping* change the data? Tape a small paper plate to the bottom of the mass and collect additional data. You may want to take data for more than 10 seconds. Does the model still fit well in this case?
3. Do additional experiments to discover the relationship between the mass and the period of this motion.